Strong-Coupling Effects in "Dirty" Superfluid ³He

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The contribution of the strong-coupling effects to the free energy of the "dirty" superfluid ³He is estimated using a simple model. It is shown that the strong-coupling effects are less susceptible to the quasiparticle scattering events in comparison to the weak-coupling counterpart. This supports the conclusion about stabilization of the B-phase in aerogel environment at pressures where the A-phase takes over in bulk superfluid ³He, in accordance with recent experimental observations in zero magnetic field.

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1. Introduction

One of the actively investigated problems in the low temperature physics is the search of the properties of a "dirty" superfluid Fermi system like liquid 3He , confined to a high-porosity aerogel environment. A number of experimental observations [1-10] revealed the new aspects of the behavior of the ordered state of the superfluid 3He in the presence of the quasiparticle scattering against a random system of silica strands forming the skeleton of aerogel.

The theoretical attempts to interpret these experiments, although partly successful, are still insufficient to describe the main body of accumulated information and the details of the phase diagram, in particular. The theoretical approach adopted up to now is based on a weak-coupling approximation. One of the conclusions obtained in this way is the claim that in a zero magnetic field the quasiparticle scattering on the spatial irregularities ("impurities") promotes the stability of the isotropic B-phase in the domain of the P-T phase diagram where in bulk ("pure") superfluid 3He the anisotropic A-phase takes over [11,12]. This theoretical result means that at pressures above the polycritical value P_{c0} the "dirty" B-phase overcomes the so called

strong-coupling effects and should appear as an equilibrium superfluid state of liquid ${}^{3}He$ confined to the aerogel environment. This conclusion is based on a supposition that the strong-coupling effects (which also are subject to the "impurity" renormalization) are less susceptible to the quasiparticle scattering events.

In what follows we explore this question in some details. It will be shown (using a simple model) that indeed it seems likely that, although the strong-coupling effects are enhanced due to the finite value of the quasiparticle mean free path, the B-phase still is able to take over the A-phase at high pressures. Quite recently, using a high frequency acoustic technique [13] the 3D phase diagram in (P, T, B) space was constructed for superfluid ${}^{3}He$ confined to 98% porosity aerogel (see, also, Ref.[14]). The covered pressure range extended from 15 bars up to the melting pressure. One of the most striking observations is that in zero magnetic field (B=0) and at all pressures above 15 bars the phase transition to the B-like phase takes part (at $T_c(P)$ with no signs of polycritical point (PCP) at wich the A- and B-phases do meet in bulk superfluid ${}^{3}He$ at P_{c0} =21 bars)). It appears that PCP for 3He in 98% porosity aerogel is absent because P_c is pushed out above the solidification pressure, and thus is unobservable. The results of our theoretical consideration seem to be in accordance with the above-mentioned experimental observations.

2. Strong-Coupling Effects in "Dirty" Superfluid ³He

The weak-coupling approach in treating the properties of superfluid phases of liquid ${}^{3}He$ disregards the inverse action of the ordering on the pairing iteraction (so called strong-coupling effects). The importance of this feedback effect is well known [15] and is mainly due to an attractive contribution of the spin exitations in the strongly correlated system to the effective quasiparticle interaction. Physically the strong-coupling feedback effect stems from the fact that the spin susceptibility of liquid ${}^{3}He$ is sensitive to the character of the spin-triplet Cooper pairing order parameter. In what follows, using a simple model, we are going to estimate the influence of the quasiparticle scattering on the strong-coupling effects having in mind an application to the superfluid ${}^{3}He$ filling the low-density aerogel. It should be stressed that a much more rafined approach in treating the strong-coupling effects is based on a systematic expansion of the free energy of superfluid ${}^{3}He$ in powers of T_c/T_F . This program has been realized in Ref.[16] (see, also, the review article [17] and Ref. [18]). This approach captures the contributions to the strong-coupling effects stemming not only from the spin fluctuations

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but from the transverse current fluctuations as well. Unfortunately, it is an extremely difficult task to treat the "impurity" effects even within relatively simple dynamical spin-fluctuation model used in Ref.[19], nothing to say about a more sophisticated approach mentioned above. Instead, we will rely on a static approximation [20] which disregardes the retardation effects in treating an attractive interaction between quasiparticales via the exchange of the "paramagnons".

We start from the momentum space Fourier component of the static spin-susceptibility tensor

$$\chi_{\mu\nu}(\vec{q}) = -\frac{1}{2}T\sum_{\omega}\sum_{k}Tr\left[\stackrel{\wedge}{G}_{\omega}(\vec{k})\stackrel{\wedge}{\sigma}_{\mu}\stackrel{\wedge}{G}_{\omega}(\vec{k}+\vec{q})\stackrel{\wedge}{\sigma}_{\nu} - \right]$$

$$\stackrel{\wedge}{\bar{F}}_{\omega}(\vec{k})\stackrel{\wedge}{\sigma}_{\mu}\stackrel{\wedge}{F}_{\omega}(\vec{k}+\vec{q})\stackrel{\wedge}{\sigma}_{\nu}^{tr}, \qquad (1)$$

where 2×2 spin-matrices \hat{G}_{ω} (\vec{k}) and \hat{F}_{ω} (\vec{k}) denote the Gorkov Green's functions (in the Matsubara representation), describing an ordered (superfluid) Fermi system. Eq.(1) can be used to construct an effective spin-depended part of the interaction potential acting between quasiparticles with the matrix elements

$$J_{\mu\nu}(\vec{k}, \vec{k'})(\stackrel{\wedge}{\sigma}_{\mu})_{\alpha\alpha'}(\stackrel{\wedge}{\sigma}_{\nu})_{\beta\beta'},\tag{2}$$

where

$$J_{\mu\nu}(\vec{k}, \vec{k'}) = -I^2 \chi_{\mu\nu}(\vec{k} - \vec{k'}) \tag{3}$$

with I standing for the local repulsive potential describing correlation effects.

In the random-phase approximation the susceptibility tensor $\overset{\wedge}{\chi}$ is constracted as

$$\stackrel{\wedge}{\chi} = \stackrel{\wedge}{\chi}^{(0)} + \stackrel{\wedge}{\chi}^{(0)} I \stackrel{\wedge}{\chi} = (\stackrel{\wedge}{1} - I \stackrel{\wedge}{\chi}^{(0)})^{-1} \stackrel{\wedge}{\chi}^{(0)}, \tag{4}$$

where $\overset{\wedge}{\chi}^{(0)}$ stands for the spin susceptibility in the absence of the correlation effects (I=0). In the vicinity of the critical temperature of the phase transition to the superfluid state

$$\stackrel{\wedge}{\chi}^{(0)} \simeq \chi_N^{(0)} \stackrel{\wedge}{1} + \delta \stackrel{\wedge}{\chi}^{(0)} \tag{5}$$

with $\chi_N^{(0)}$ being the normal-state susceptibility. The superfluid contribution $\delta \stackrel{\wedge}{\chi}^{(0)}$ is quadratic in the order parameter of the superfluid state. Consequently, at $T \stackrel{<}{\sim} T_c$

$$J_{\mu\nu} \simeq -\frac{I^2 \chi_N^{(0)}}{1 - I \chi_N^{(0)}} \delta_{\mu\nu} - \left(\frac{I}{1 - I \chi_N^{(0)}}\right)^2 \delta \stackrel{\wedge}{\chi}_{\mu\nu}^{(0)} . \tag{6}$$

The results concerning the strong-coupling contribution in 3He and based on the mentioned simple model are described in Ref.[21]. They show that this model reflects the essence of the feedback effects at least qualitatively. At the same time, in the framework of the adopted static model the technical side of the calculation of the quasiparticle scattering effects (which is our main goal) is simplified considerably. Hopefully the static model, which we adopted, gives at least qualitatively meaningful treatment of the relative stability of the A- and B-phases of a "dirty" superfluid 3He confined to aerogel environment in zero magnetic field.

In order to estimate the effects of the finite mean free path of the quasiparticles we address a self-consistancy equation for the order parameter matrix $\overset{\wedge}{\Delta}$ which in the case of a spin-triplet Cooper pairing (appropriate to the superfluid 3He) is given as

$$\stackrel{\wedge}{\Delta} = \Delta_{\mu} (\stackrel{\wedge}{\sigma}_{\mu} i \stackrel{\wedge}{\sigma}_{y}). \tag{7}$$

The equation for the vector component Δ_{μ} reads as

$$\Delta_{\mu}(\hat{k}) = -T \sum_{\omega} \sum_{k'} V_{\mu\nu}(\vec{k}, \vec{k'}) F_{\omega\nu}(\vec{k'}), \tag{8}$$

where \hat{k} is the unit vector along the momentum \vec{k} ,

$$V_{\mu\nu} = V\delta_{\mu\nu} + \delta V_{\mu\nu},\tag{9}$$

and the feedback contribution $\delta V_{\mu\nu} = J_{\lambda\lambda}\delta_{\mu\nu} - 2J_{\mu\nu}$. In eq.(8) $F_{\omega\nu}$ denotes the ν -th vector component connected to \hat{F}_{ω} in a way similar to Eq.(7). In the adopted model

$$\delta V_{\mu\nu} = -\left(\frac{I}{1 - I\chi_N^{(0)}}\right)^2 (\delta\chi_{\lambda\lambda}^{(0)}\delta_{\mu\nu} - 2\delta\chi_{\mu\nu}^{(0)}). \tag{10}$$

In terms of the quasiclassical function

$$f_{\omega\nu}(\hat{k}) = \frac{1}{\pi} \int_{-\infty}^{+\infty} d\xi F_{\omega\nu}(\hat{k}, \xi) \tag{11}$$

the self-consistency Eq.(8) reads as

$$\Delta_{\mu}(\hat{k}) = 2\pi T \sum_{\omega > 0} \sum_{k'} \langle 3\hat{k}\hat{k'}g_{\mu\nu}f_{\omega\nu}(\hat{k'})\rangle, \tag{12}$$

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where the brackets denote the averaging over the orientation of \hat{k}' and $g_{\mu\nu}$ stands for an attractive component in the p-wave channel. According to our consideration $g_{\mu\nu} = g\delta_{\mu\nu} + \delta g_{\mu\nu}$ with $\delta g_{\mu\nu}$ stemming from the spin-dependent part of the quasiparticle interaction (see Eq.(10)).

For the case of bulk superfluid 3He near T_{c0} (the critical temperature of a pure system) and at $q \ll k_F$

$$\delta \chi_{\mu\nu}^{(0)}(\vec{q}) = -2\pi T \sum_{\omega > 0} \frac{N_F}{2\omega^3} \left(\frac{2\omega}{qv_F} \right)^2 \left\langle \frac{\text{Re}(\Delta_{\mu}(\hat{\mathbf{k}}) \Delta_{\nu}^*(\hat{\mathbf{k}}))}{(\hat{k}\hat{q})^2 + (2\omega/qv_F)^2} \right\rangle, \tag{13}$$

where N_F and v_F denote the density of states and the velocity of the quasiparticles at the Fermi level. After averaging over the orientation of \hat{q} , from Eq.(13) it is obtained that

$$\delta\chi_{\mu\nu}^{(0)}(q) = -\frac{N_F}{qv_F} \left[2\pi T \sum_{\omega>0} \frac{1}{\omega^2} \arctan(qv_F/2\omega) \right] \operatorname{Re}\langle \Delta_{\mu}(\hat{\mathbf{k}}) \Delta_{\nu}^*(\hat{\mathbf{k}}) \rangle. \tag{14}$$

The main contribution to the feedback coupling constant $\delta g_{\mu\nu}$ is to be extracted from the region of $q \gg \xi_{c0}^{-1}$ where the coherence length $\xi_{c0} = v_F/2\pi T_{c0}$. In this limit Eq.(14) gives

$$\delta\chi_{\mu\nu}^{(0)}(q) = -\frac{\pi}{2} \frac{N_F}{q v_F} \left(2\pi T \sum_{\nu > 0} \frac{1}{\omega^2} \right) \operatorname{Re}\langle \Delta_{\mu}(\hat{\mathbf{k}}) \Delta_{\nu}^*(\hat{\mathbf{k}}) \rangle \tag{15}$$

and as a result

$$\delta g_{\mu\nu} = \frac{\delta g}{(\pi T_{c0})^2} \langle |\vec{\Delta}|^2 \delta_{\mu\nu} - \Delta_{\mu} \Delta_{\nu}^* - \Delta_{\mu}^* \Delta_{\nu} \rangle, \tag{16}$$

with

$$\delta g = \frac{1}{6} \left(\frac{\pi}{2}\right)^3 \left(\frac{IN_F}{1 - IN_F}\right)^2 \frac{1}{k_F \xi_{c0}}.$$
 (17)

Our main concern is to establish the modification of Eq.(16) caused by the quasiparticle scattering against the irregularities ("impurities") introduced by the presence of aerogel silica stands. The spin susceptibility is a two-particle correlator and the corresponding system of equations is to be addressed. The impurity scattering effects show up, in particular, as the vertex corrections complicating considerably the general consideration. Fortunately at $\xi_c^{-1} \ll q \ll k_F$, which is the region of the momentum transfer we are interested in, the vertex corrections are small as far as $(k_F \xi_c)^{-1} \ll 1$. Finally, the essential contribution to the feedback effect modification due to the scattering events are simply realized by the substitution $\omega \to \tilde{\omega} = \omega + \Gamma sgn \omega$ in

Eq.(15), where $\Gamma = \frac{c}{\pi N_F} \sin^2 \delta_0$ is the quasiparticles scattering rate (in what follow we adopt the so called homogeneus scattering model (HSM) with the s-wave scattering channel only (see Refs.[11,12]). Here c denotes the "impurity" concentration and δ_0 is the phase shift at an s-wave scattering. As a result the coupling constant $\delta g_{\mu\nu}$ (see Eq.(16)) is transformed to

$$\delta \tilde{g}_{\mu\nu} = \frac{\delta \tilde{g}}{(\pi T_c)^2} \langle |\vec{\Delta}|^2 \delta_{\mu\nu} - \Delta_{\mu} \Delta_{\nu}^* - \Delta_{\mu}^* \Delta_{\nu} \rangle, \tag{18}$$

where T_c stands for the critical temperature of the phase transition of liquid 3He in aerogel to an ordered state and

$$\delta \tilde{g} = \frac{1}{6} \left(\frac{\pi}{2}\right)^3 \left(\frac{IN_F}{1 - IN_F}\right)^2 \frac{1}{k_F \xi_c} \frac{\psi^{(1)}(1/2 + w)}{\pi^2/2}, \quad \xi_c = \frac{v_F}{2\pi T_c}.$$
 (19)

Here $\psi^{(m)}(z)$ denotes the poly-gamma function and $w = \Gamma/2\pi T$.

Taking into account that up to the third order in $\vec{\Delta}$ and in the presence of the quasiparticle scattering centers

$$\vec{f}_{\omega} \simeq \frac{\vec{\Delta}}{|\tilde{\omega}|} - \frac{1}{2|\tilde{\omega}|^3} \left[(\vec{\Delta}^* \vec{\Delta}) \vec{\Delta} + (\vec{\Delta}^* \times \vec{\Delta}) \times \vec{\Delta} - \frac{\Gamma \cos 2\delta_0}{|\tilde{\omega}|} \left(\langle |\vec{\Delta}|^2 \rangle \vec{\Delta} + \langle (\vec{\Delta}^* \times \vec{\Delta}) \rangle \times \vec{\Delta} \right) \right], \tag{20}$$

after averaging over the orientation of \hat{k}' in the self-consistency Eq.(12), the equation for the order parameter $\vec{\Delta}(\hat{k})$ near T_c reads as (for the superfluid 3He $\Delta_{\mu}(\hat{k}) = A_{\mu i}\hat{k}_i$):

$$(a_{1}(T) - 1/g)\vec{\Delta}(\hat{k}) = \frac{3}{5}a_{3}\left(-\frac{1}{2}\langle\vec{\Delta}^{2}\rangle\vec{\Delta}^{*}(\hat{k}) + \langle|\vec{\Delta}|^{2}\rangle\vec{\Delta}(\hat{k}) + (21)\right)$$

$$\langle\vec{\Delta}\Delta_{\nu}\rangle\Delta_{\nu}^{*}(\hat{k}) + \langle\vec{\Delta}\Delta_{\nu}^{*}\rangle\Delta_{\nu}(\hat{k}) - \langle\vec{\Delta}^{*}\Delta_{\nu}\rangle\Delta_{\nu}(\hat{k})\right) - \frac{1}{2}\Gamma\cos2\delta_{0}a_{4}\left(\langle|\vec{\Delta}|^{2}\rangle\vec{\Delta}(\hat{k}) + \langle\vec{\Delta}\Delta_{\nu}^{*}\rangle\Delta_{\nu}(\hat{k}) - \langle\vec{\Delta}^{*}\Delta_{\nu}\rangle\Delta_{\nu}(\hat{k}) + \frac{\delta\tilde{g}}{g}\frac{a_{1}}{(\pi T_{c})^{2}}\left(\langle|\vec{\Delta}|^{2}\rangle\vec{\Delta}(\hat{k}) - \langle\vec{\Delta}\Delta_{\nu}^{*}\rangle\Delta_{\nu}(\hat{k}) - \langle\vec{\Delta}^{*}\Delta_{\nu}\rangle\Delta_{\nu}(\hat{k})\right),$$

where

$$a_1(T) = 2\pi T \sum_{\omega > 0}^{\omega_c} \frac{1}{\tilde{\omega}} = \ln\left(\frac{2\gamma}{\pi} \frac{\omega_c}{T}\right) + \psi(1/2) - \psi(1/2 + w), \tag{22}$$

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$$a_3(T) = 2\pi T \sum_{\omega > 0} \frac{1}{\tilde{\omega}^3} = -\frac{1}{2} \frac{1}{(2\pi T)^2} \psi^{(2)}(1/2 + w),$$
 (23)

$$a_4(T) = 2\pi T \sum_{\omega > 0} \frac{1}{\tilde{\omega}^4} = \frac{1}{6} \frac{1}{(2\pi T)^3} \psi^{(3)}(1/2 + w).$$
 (24)

Finally, for the order parameter $A_{\mu i}$ the following equation is obtained from Eq.(21)

$$\alpha(T)A_{\mu i} + \frac{N_F}{15}a_3 \left\{ -\frac{1}{2}A_{\mu i}^* A_{\nu j} A_{\nu j} + \left[\left(1 - \frac{5}{6}\Gamma\cos 2\delta_0 \frac{a_4}{a_3} \right) + \delta_{sc} \right] A_{\mu i} A_{\nu j}^* A_{\nu j} + A_{\mu j} A_{\nu j} A_{\nu i}^* + \left[\left(1 - \frac{5}{6}\Gamma\cos 2\delta_0 \frac{a_4}{a_3} \right) - \delta_{sc} \right] A_{\mu j} A_{\nu j}^* A_{\nu i} - \left[\left(1 - \frac{5}{6}\Gamma\cos 2\delta_0 \frac{a_4}{a_3} \right) + \delta_{sc} \right] A_{\mu j}^* A_{\nu j} A_{\nu i} \right\} = 0,$$
(25)

where $\alpha(T) = \frac{1}{3}N_F \left[\ln \frac{T}{T_{c0}} + \psi(1/2 + \omega) - \psi(1/2) \right]$ and the strong-coupling contribution is described by a parameter

$$\delta_{sc} = \frac{5}{3} \frac{\delta \tilde{g}}{q^2} \frac{1}{a_3} \frac{1}{(\pi T_c)^2}.$$
 (26)

In Eqs.(25) and (26) the coefficients a_3 and a_4 are to be calculated at $T = T_c$. Comparing Eq.(25) with its phenomenological Ginzburg-Landau counterpart

$$\alpha(T)A_{\mu i} + 2(\beta_1 A_{\mu i}^* A_{\nu j} A_{\nu j} + \beta_2 A_{\mu i} A_{\nu j}^* A_{\nu j} + \beta_3 A_{\mu j} A_{\nu i}^* + \beta_4 A_{\mu j} A_{\nu j}^* A_{\nu i} + \beta_5 A_{\mu j}^* A_{\nu j} A_{\nu i}) = 0,$$
(27)

the β -coefficients can be identified. Representing β_i as the sum of week-coupling (β_i^{wc}) and strong-coupling $(\delta\beta_i^{sc})$ contributions, it is found that

$$-2\beta_1^{wc} = \beta_3^{wc} = 2\beta_{wc} = \frac{7\zeta(3)}{120} \frac{N_F}{(\pi T_c)^2} \frac{\psi^{(2)}(1/2 + w_c)}{\psi^{(2)}(1/2)},$$
 (28)

$$\beta_2^{wc} = \beta_4^{wc} = -\beta_5^{wc} = 2\beta_{wc} - \frac{\Gamma \cos 2\delta_0}{12^3} \frac{N_F}{(\pi T_c)^3} \psi^{(3)}(1/2 + w_c), \tag{29}$$

$$\delta \beta_1^{sc} = \delta \beta_3^{sc} = 0, \qquad (30)$$

$$\delta\beta_2^{sc} = -\delta\beta_4^{sc} = -\delta\beta_5^{sc} = \delta\beta_{sc},\tag{31}$$

$$\delta\beta_{sc} = \frac{1}{2(4g)^2} \left(\frac{\pi}{3}\right)^3 \frac{N_F}{(\pi T_c)^2} \left(\frac{IN_F}{1 - IN_F}\right)^2 \frac{1}{k_F \xi_c} \frac{\psi^{(1)}(1/2 + w_c)}{\psi^{(1)}(1/2)},\tag{32}$$

where $w_c = \frac{\Gamma}{2\pi T_c}$. It can be easily verified that the weak-coupling coefficients β_i^{wc} reproduce the answer reported in Ref.[11].

In order to explore the domain of the phase diagram (in the Ginzburg-Landau region) where the B-phase overcomes the strong-coupling effects and is preferable as an equilibrium state in comparision to the A-phase (in zero magnetic field), we address a well known inequality

$$\beta_{12} + \frac{1}{3}\beta_{345} < \beta_{245}. \tag{33}$$

Introducing the normalized β -coefficients $\bar{\beta}_i = \beta_i/|\beta_1^{wc}|$, the criterion of thermodynamical stability of the B-phase in the Ginzburg-Landau region reads as

$$-2\delta\bar{\beta}_{345}^{sc} + 3\delta\bar{\beta}_{13}^{sc} < 1. \tag{34}$$

According to Eq.[28] $|\beta_1^{wc}| = \beta_{wc}^0 R_{wc}$ where the "impurity" renormalization factor for the weak-coupling coefficient β_{wc} is given by

$$R_{wc}(w_c) = \frac{\psi^{(2)}(1/2 + w_c)}{\psi^{(2)}(1/2)} \left(\frac{T_{c0}}{T_c}\right)^2.$$
 (35)

On the other hand, following Eq.(32) $\delta\beta_{sc}=\delta\beta_{sc}^0R_{sc}$ with the "impurity" renormalization factor

$$R_{sc}(w_c) = \frac{\psi^{(1)}(1/2 + w_c)}{\psi^{(1)}(1/2)} \frac{T_{c0}}{T_c}.$$
 (36)

It is to be remembered that the ratio $T_{c0}/T_c(w_c)$ is found from the Abrikosov-Gorkov type equation

$$\ln(T_{c0}/T_c) + \psi(1/2) - \psi(1/2 + \omega_c) = 0.$$
(37)

The renormalization factors R_{wc} and R_{sc} are monotonously increasing functions of w_c although the strong-coupling effects are less susceptible to the quasiparticle scattering events.

Collecting these results, in the framework of the adopted simple model the B-phase stability region near T_c is defined by the inequality

$$\delta \bar{\beta}_{sc} = \delta \bar{\beta}_{sc}^{0}(P)R(w_{sc}) < \frac{1}{4}, \tag{38}$$

where the effects of the finite mean free path of the quasiparticles in aerogel environment are accumulated in the renormalization factor

$$R(w_c) = \frac{R_{sc}(w_c)}{R_{wc}(w_c)} = a(w_c) \frac{T_c}{T_{c0}}$$
(39)

with

$$a(w_c) = \frac{\psi^{(1)}(1/2 + w_c)}{\psi^{(1)}(1/2)} \frac{\psi^{(2)}(1/2)}{\psi^{(2)}(1/2 + w_c)}.$$
 (40)

It is to be noted that in Eq.(38) the presence of T_c/T_{c0} certainly stems from the fact that the strong-coupling corrections to the free energy contain an extra powers in T_c/T_F (in comparison with the weak-coupling contribution). At the same time, Eq.(38) shows that $R(w_c)$ is not simply equal to T_c/T_{c0} but contains an extra factor $a(w_c)$ which increases with the quasiparticle scattering rate and competes with T_c/T_{c0} which decreases with w_c . The analyses shows that this competition is in favor of T_c/T_{c0} and $R_{wc} < 1$ at $w_c > 0$. In particular, for the case with $w_c < 1$ at $w_c > 1 - \frac{1}{2}\pi^2w_c$, so that $R(w_c) \simeq 1 - 2.56w_c$.

Turning back to Eq.(37) it is concluded that, since in the quasiparticle scattering medium $R(w_c) < 1$, the condition of the stability of the B-phase in aerogel is less restrictive in comparison with bulk case. This opens a way for the appearence of the B-like superfluid state in the pressure region $P > P_{c0}$ which increases with the quasiparticle scattering intensity. As was mentioned in the Introduction, in case of 98% porosity aerogel the B-like phase near T_c and in zero magnetic field is observed up to the melting pressure P_m . For larger porosity aerogel (with smaller w_c) the PCP may appear in the pressure region $P_{c0} < P < P_m$, as mentioned in Ref.[13].

3. Conclusion

As is well known, an isotropic B-phase of superfluid ${}^{3}He$ is stabilized at pressures below $P_{c0} \simeq 21$ bars. At higher pressures the A-phase takes over due to the strong-coupling effects manifested as a feedback of the Cooper pairing on the quasiparticle attractive interaction.

On the other hand, in recent experimental studies (see Ref.[13]) of the phase diagram of a "dirty" superfluid ${}^{3}He$ confined to the 98% porosity silica aerogel, it was established that a B-phase-like ordered state is stabilized at $P > P_{c0}$ up to $P = P_{m}$. This observation indicates that the scattering of quasiparticles against the spatial irregularities of the porous medium modifies the free energy of superfluid ${}^{3}He$ in favor of the B-phase as an equilibrium ordered state at high pressures. The free energy of superfluid state can be viewed as containing the two contributions stemming from the weak-coupling and strong-coupling effects. The former contribution for the "dirty" superfluid ${}^{3}He$ has been investigated theoretically in Ref.[11] where it was shown that the "impurity" renormalization of the week-coupling part

of the free energy (near T_c) promotes the stabilization of the B-phase at the pressures where in bulk superfluid 3He the A-phase is an equilibrium ordered state. This conclusion, as mentioned in Ref.[11], supposes that the strong-coupling effects are not more susceptible to the quasiparticle scattering then their weak-coupling counterpart. In using a simple model to treat the strong-coupling contribution to the free energy, we have demonstrated that the "impurity" renormalization of this contribution is considerably weaker in comparison to the weak-coupling effects. This conclusion seems to be in accordance with the mentioned experimental observations.

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